

Revisiting the Far-field Model of the Force and Torque Induced on a Conductive Nonmagnetic Sphere by a Rotating Magnetic Dipole: Supplementary Materials

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Comparison to prior model at configurations $\theta = 0^\circ$ and $\theta = 90^\circ$

In Fig. S1, we compare the ψ values resulting from the data collected in this study, using the model coefficients of Table 1, to those resulting from the model coefficients provided in Pham *et al.*⁹, which are included in Table S1 along with their associated R_{adj}^2 values. We only compare results at configurations $\theta = 0^\circ$ and $\theta = 90^\circ$, since those were the only ones considered previously.

Table S1. Component-specific model coefficients of the prior model by Pham *et al.*⁹ The finite-element-analysis (FEA) coefficients and experimental coefficients were determined over the domains $\Pi_1 \in [0, 20.1]$ and $\Pi_1 \in [0, 3.65]$, respectively.

Coefficients at $\theta = 0^\circ$											
	ξ	FEA Coefficients					Experimental Coefficients				
		c_0	c_1	c_2	c_3	R_{adj}^2	c_0	c_1	c_2	c_3	R_{adj}^2
f_ρ	-7	430	2.95	-0.101	-9.26	0.969	467	2.81	-0.0969	-9.75	0.996
τ_ρ	-6	6840	3.00	-0.0986	-13.2	0.974	6900	3.35	-0.0990	-14.9	0.999

Coefficients at $\theta = 90^\circ$											
	ξ	FEA Coefficients					Experimental Coefficients				
		c_0	c_1	c_2	c_3	R_{adj}^2	c_0	c_1	c_2	c_3	R_{adj}^2
f_ρ	-7	266	2.60	-0.101	-7.65	0.981	282	3.20	-0.0980	-9.41	0.919
f_ϕ	-7	6040	3.45	-0.102	-14.3	0.963	5870	3.49	-0.0973	-14.6	0.997
τ_θ	-6	8100	3.60	-0.0985	-15.7	0.928	8000	3.40	-0.0928	-15.0	0.999

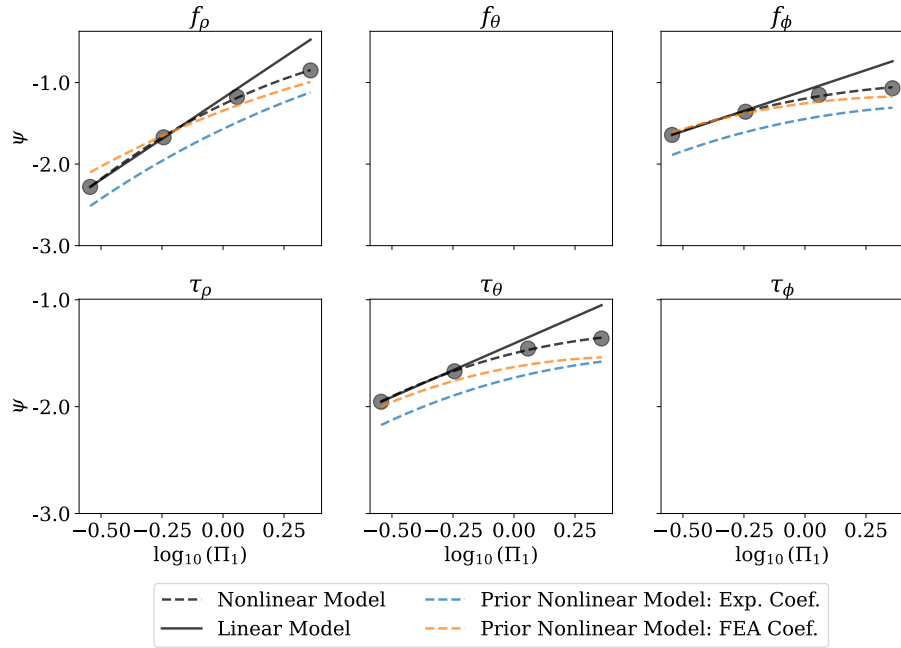
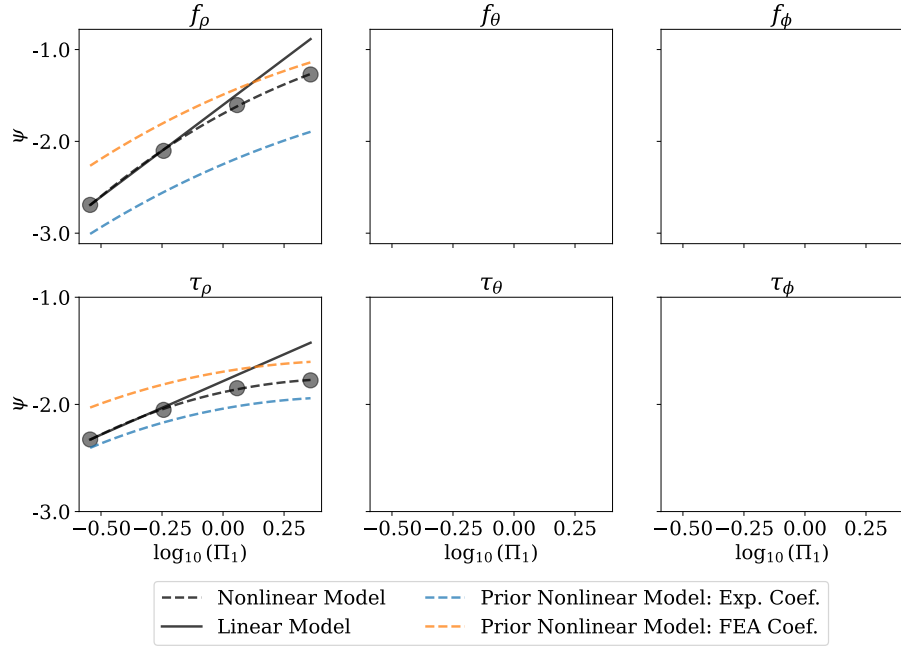


Figure S1. Comparison of the experimentally obtained values of ψ and the nonlinear and linear models of ψ from this study with the prior nonlinear model of ψ from Pham *et al.*,⁹ which uses two different sets of coefficients: those generated from experimental trials, and those generated from finite-element-analysis (FEA) trials (see Table S1).

In-plane contribution of f_θ

In the plane spanned by $\boldsymbol{\rho}$ and $\boldsymbol{\omega}$, the object experiences a force comprising components f_ρ and f_θ , which are both quadratically proportional to ω . In the prior model, f_θ was assumed to be negligible compared to f_ρ . As a result of modeling at configuration $\theta = 45^\circ$, its in-plane contribution can be analyzed, and quantified by

$$\begin{aligned} \text{Contribution of } f_\theta &:= \frac{|f_\theta|}{\sqrt{f_\rho^2 + f_\theta^2}} \\ &= \frac{10^{\beta(\theta, 45^\circ)} \sin(2\theta)}{\sqrt{\frac{1}{4} [10^{\beta(\rho, 0^\circ)} (1 + \cos(2\theta)) + 10^{\beta(\rho, 90^\circ)} (1 - \cos(2\theta))]^2 + [10^{\beta(\theta, 45^\circ)} \sin(2\theta)]^2}} \end{aligned} \quad (\text{S1})$$

which is invariant to both Π_1 and Π_2 , where the interpolation functions of (5) enable analysis at all values of θ . The contribution of f_θ is depicted in Fig. S2, with a maximal value of 16.7%, occurring symmetrically at $\theta = 32^\circ$, and $\theta = 148^\circ$. The values of coefficient β in (S1) are denoted by: $\beta(\theta, 45^\circ)$ for the f_θ component at configuration $\theta = 45^\circ$, and $\beta(\rho, 0^\circ)$ and $\beta(\rho, 90^\circ)$ for the f_ρ component at configurations $\theta = 0^\circ$ and $\theta = 90^\circ$, respectively (see Table 1).

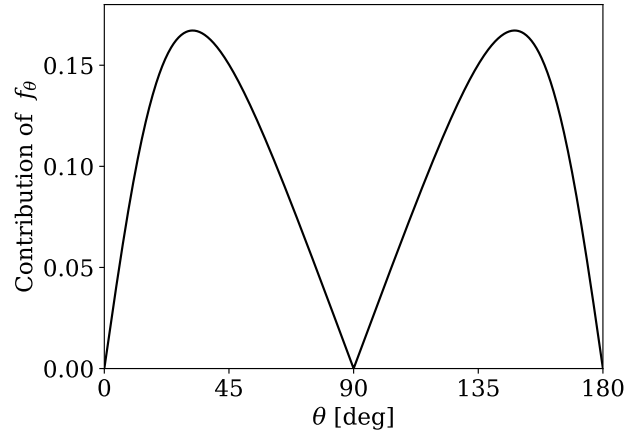


Figure S2. The contribution of force component f_θ to the force vector induced in the plane spanned by vectors $\boldsymbol{\rho}$ and $\boldsymbol{\omega}$ as a function of θ .