

Electromagnetics

Assume a Spherical Cube: Bounding the Force and Torque Induced on a Conductive Nonmagnetic Cube by a Rotating Magnetic Dipole

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Abstract—A time-varying magnetic field induces electric currents in a conductive object, which in turn induces a force–torque wrench on the object that is not due to ferromagnetism. Prior work empirically modeled the wrench induced in a solid sphere by a rotating magnetic dipole (RMD) field, and then used this model to perform contactless manipulation using multiple RMD field sources, motivated by application in on-orbit satellite servicing and space-debris capture. In this letter, we measure the induced wrench on a conductive, nonferromagnetic cube—which, unlike on a sphere, is not invariant to orientation—in six canonical configurations, and we compare it to that of a sphere with identical volume. Results show that such a spherical model provides a reasonable orientation-invariant approximation of a cube. Further, the induced wrench can be bounded by considering the induced wrenches on spheres with $\pm 15\%$ volume.

Index Terms—Electromagnetics, electromagnetic induction, force, modeling, torque.

I. INTRODUCTION

Time-varying magnetic fields induce electric currents in a conductive object. These eddy currents interact with the applied magnetic field, resulting in an induced force and/or torque on the object [Hertz 1880]. An emerging area of interest for this phenomenon is on-orbit servicing of satellites and capture of space debris, with a particular emphasis on despinning/detumbling uncooperative resident space objects (RSOs) [Sugai 2013, Gómez 2015, Yongquist 2016, Nurge 2017, Liu 2018, Allen 2024, Abbott 2025]. Contactless manipulation is desirable because it has the benefit of reducing the chance of a collision that could damage the RSO or create more space debris. However, traditional contactless manipulation via ferromagnetism is infeasible, as RSOs largely comprise aluminum [Opiela 2009].

Our group has shown that the use of induced forces and torques to manipulate conductive, nonferromagnetic objects can be formulated as a robotic manipulation problem, where controlled manipulation in six degrees of freedom (6-DOF) is achievable using two or more field sources that each generate a controllable rotating magnetic dipole (RMD) field [Pham 2021, Dalton 2022, 2024, Tabor 2025]. This dexterous manipulation has been enabled by a model of the induced force–torque wrench on solid conductive spheres—originally formulated by Pham [2021] and later expanded on by Barker [2025] and Tabor [2025]—which can serve as a first-order approximation for other object geometries. Tabor [2025] showed that dexterous manipulation can be extended to nonspherical objects by using the spherical model in an adaptive control framework where the sphere’s radius and conductivity are updated online to identify a spherical approximation that best describes the immediate observed behavior of the nonspherical

object. With sufficient time, the tracking performance of the controller is robust to a poor initial estimate of the spherical approximation; however, the initial performance of the controller is sensitive to a poor parameter initialization. The high-consequence nature of on-orbit servicing necessitates a procedure for quickly determining an initial parameter set for a given object so that controller performance converges quickly.

Further, many RSOs have angular momentum due to environmental disturbances and/or lack of attitude control, causing them to tumble about arbitrary axes. To safely capture an RSO, some of this residual angular momentum must be transferred to the servicer craft to stabilize the RSO’s relative orientation [Shan 2016]. Many RSOs exhibit nonuniform shapes, and, in such a detumbling procedure, it is unknown by how much the induced force and torque will change as the orientation of the RSO varies with respect to the servicer craft’s RMD field source(s).

To address these challenges, in this letter, we present a study in which we measured the mean induced wrench on a conductive, nonferromagnetic cube in six canonical configurations relative to an RMD field. We are interested in the cube because it is a common satellite shape, and it can also approximate other nonspherical geometries. Unlike with a sphere, changing a cube’s orientation changes the distribution of the conductive material within the RMD field and, thus, the induced wrench. We then used the model of Barker [2025] to compute the wrench that would be induced on a sphere whose volume is equivalent to that of the cube, or some multiple of that volume, resulting in a bounding model for a cube.

II. METHODS

Our methods follow those used in the development of our state-of-the-art model of the induced wrench on spheres [Barker 2025]. We use the spherical coordinate system of Fig. 1(a) to describe the position

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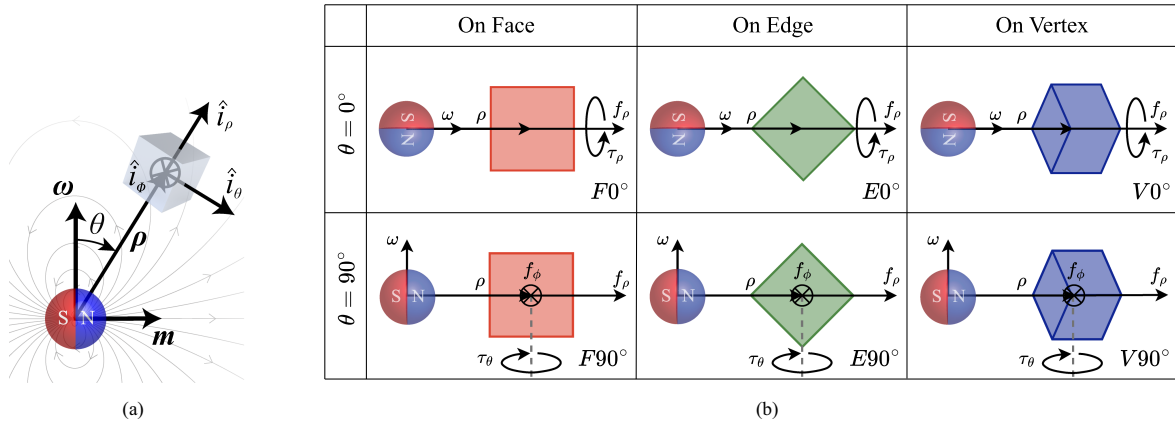


Fig. 1. Schematic of induced forces and torques on a conductive cube. (a) Definition of the spherical coordinate system used to parameterize cube location and force/torque vectors with respect to an RMD field source. The magnetic dipole m and its field are depicted at a given instant, but are not relevant to the definition of the coordinate system. The cube is depicted with an arbitrary orientation. (b) Definition of three canonical orientations of a cube with respect to an RMD, with the name of each cube's configuration provided in the lower right of the respective image. The induced force/torque components are depicted, with all force/torque arrowheads showing both the positive sign convention and the actual directions for the ω shown. Note: $\hat{i}_\phi$ and f_ϕ point into the page, denoted symbolically by $\otimes$.

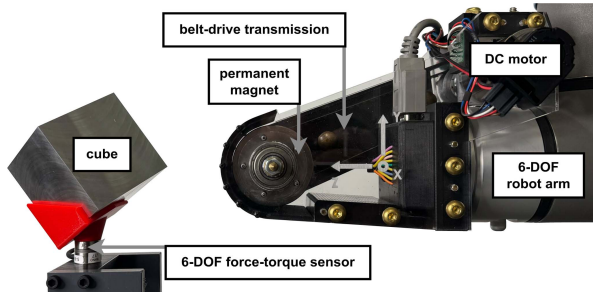


Fig. 2. Experimental apparatus shown at configuration $V90^\circ$, with ω pointing out of the page.

of the cube with respect to the RMD field source (parameterized by the distance $\rho = \|\rho\|$ and the polar angle θ) and to parameterize the induced force and torque vectors. We consider six canonical configurations of a cube with respect to an RMD field source, as shown in Fig. 1(b). These configurations correspond to two canonical values of θ , namely, $\theta = 0^\circ$ and $\theta = 90^\circ$, and three canonical cube orientations in which ρ passes through the centroid of a face of the cube (denoted F), the midpoint of an edge of the cube (denoted E), and a vertex of the cube (denoted V).

Our experimental apparatus is shown in Fig. 2. We used a solid cube of 2011-T3 aluminum with edge length 61.22 mm and electrical conductivity $2.103 \times 10^7 \text{ S} \cdot \text{m}^{-1}$. The wrench induced on this cube was measured using a 6-DOF force–torque sensor (ATI Nano17 Titanium with calibration SI-8-0.05), which collects data at a rate of 1 kHz. The cube was rigidly mounted to the sensor with a 3-D-printed adapter, and the sensor was fixed to a support structure fabricated from nonconductive materials. Mounted to a 6-DOF robot arm (Universal Robots UR5e) is the end-effector developed by Allen [2024], which generates a time-varying magnetic field via rotation of a diametrically magnetized cylindrical permanent magnet with dipole-moment magnitude $m = 102.5 \text{ A} \cdot \text{m}^2$.

We collected data at $\rho \in \{12, 13, 14, 15, 16\}$ cm at a fixed frequency of $\omega = \|\omega\| = 30 \text{ Hz}$, and at frequencies of $\omega \in \{3.75, 7.5, 15, 30, 60\} \text{ Hz}$ at a fixed $\rho = 12 \text{ cm}$. The minimum value of ρ was chosen to preserve the accuracy of the point dipole

approximation, valid for distances exceeding 1.5 radii of the magnet's minimum bounding sphere [Petruska 2013], and the maximum value of ρ was chosen to ensure a sufficient signal-to-noise ratio based on pilot studies. The minimum value for ω was likewise constrained by signal-to-noise considerations, and the maximum value was set by the capability of the apparatus.

For each combination of experimental parameters, we performed the following procedure to experimentally determine each component of force and torque.

- 1) The end-effector was placed by the robot at the pose specified by the trial's θ and ρ , using the MoveIt 2 motion planner.
- 2) 6-DOF force–torque measurements were collected for 5 s while the end-effector's permanent magnet was held still; these data were time-averaged.
- 3) 6-DOF force–torque measurements were collected for 10 s while the permanent magnet accelerated to the ω specified for each trial, where it was maintained under closed-loop control. The final 5 s of data, representing steady-state behavior, were averaged. The permanent magnet was then brought to rest.
- 4) Sensor bias was removed by subtracting the results of Step 2 from the results of Step 3. These results were then converted from the sensor's coordinate frame to the cube's body-centered coordinate frame and then expressed in the spherical coordinate system of the RMD field source.
- 5) Steps 2 through 4 were repeated, for a total of five trials per parameter combination.

We used the general model of Barker [2025] to compute the wrench that would be experienced by a sphere with the same volume as the cube used in the aforementioned experiments. In addition, we used the general model to find a sphere of some volume that would have resulted in the same magnitude of the given wrench component.

III. RESULTS AND DISCUSSION

In Figs. 3 and 4, we show experimental results for the induced force/torque components as a function of ρ (at fixed ω) and as a function of ω (at fixed ρ), respectively. Calculated force–torque

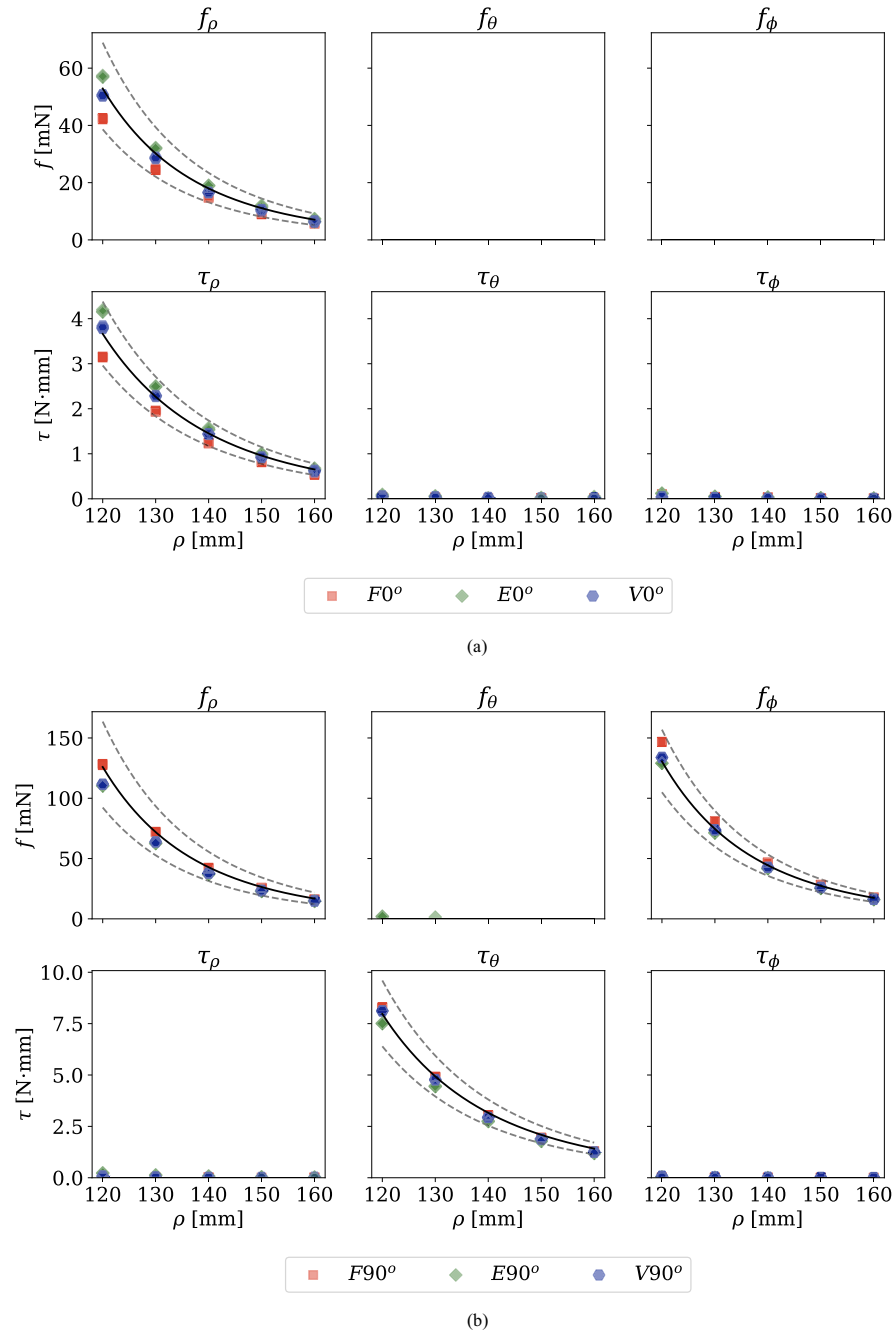


Fig. 3. Induced force and torque components as a function of ρ , for a constant $\omega = 30$ Hz, at configurations (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$. Experimental data are shown as colored pips whose coloration and shape are detailed in Fig. 1(b). All data exceeding the sensor's published resolution are shown. Calculated force and torque for a sphere of equivalent volume is shown as a solid black line; calculated force and torque for spheres of $\pm 15\%$ equivalent volume are shown as dashed lines. Calculated forces and torques are shown for all parameters that belong to the domain on which the general model of Barker [2025] was fit.

components on a sphere of equivalent volume, as well as spheres of $\pm 15\%$ volume, are shown for all parameters that belong to the domain on which the general model of Barker [2025] was fit; we do not extrapolate the model. The entire recorded dataset falls between volume multiples [0.854, 1.15]. Of the 270 individual trials, 194 (i.e., 72%) fall between volume multiples [0.9, 1.1]. For certain sets of experimental parameters, we see a substantial effect of cube orientation on the variation in componentwise force–torque magnitude, whereas for others, the effect of cube orientation is much smaller. Although the componentwise force–torque magnitude varies

with the cube's orientation at each pair of ρ and ω , the range of measured values always includes that which would be induced on a sphere of equivalent volume. Because the three orientations considered are literal and figurative corner and edge cases of the manifold of possible orientations, we expect the induced wrench at any given orientation to be bounded by those in the orientations considered here.

The ordering of the componentwise force–torque magnitudes is not consistent for the two values of θ at which data were recorded. In Figs. 3(a) and 4(a), which show the data corresponding to $\theta = 0^\circ$, we

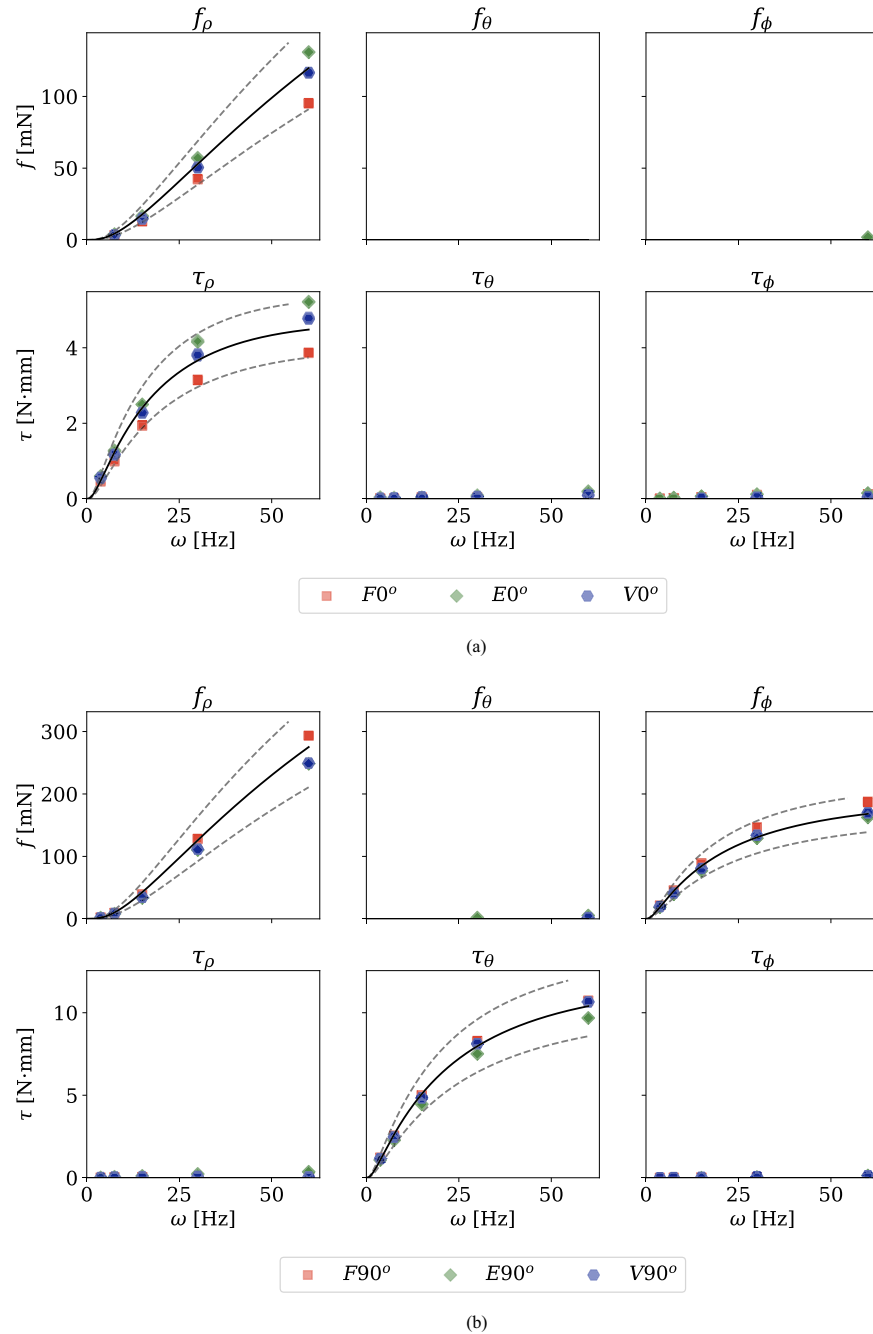


Fig. 4. Induced force and torque components as a function of ω , for a constant $\rho = 12$ cm, at configurations (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$. Experimental data are shown as colored pips whose coloration and shape are detailed in Fig. 1(b). All data exceeding the sensor's published resolution are shown. Calculated force and torque for a sphere of equivalent volume is shown as a solid black line; calculated force and torque for spheres of $\pm 15\%$ equivalent volume are shown as dashed lines. Calculated forces and torques are shown for all parameters that belong to the domain on which the general model of Barker [2025] was fit.

see, that across all values of ρ and ω , the cube with the E orientation experiences the largest componentwise force–torque magnitude, followed by the cube with the V orientation, followed by the cube with the F orientation. In contrast, the results in Figs. 3(b) and 4(b), which show the data recorded at $\theta = 90^\circ$, show that the descending force–torque magnitude order is F , followed by V , followed by E . In all cases, the E and F orientations define the bounds, never the V orientation. Although the results of our study were achieved through experiments using dimensioned quantities, the nondimensional parametric modeling of Pham [2021] ensures that they are not anecdotal and

that they generalize to cubes of any size and any electrical conductivity. This was experimentally verified in pilot studies.

Given the results above taken in their totality, we recommend that a good (initial) spherical approximation of a nonspherical object is a sphere of equivalent volume. Further, we propose the volume multiples of $[0.85, 1.15]$ (i.e., $\pm 15\%$) as a means of bounding the uncertainty in the induced wrench. As a point of comparison, the cube's insphere volume corresponds to a volume multiple of 0.525, and the cube's circumsphere volume corresponds to a volume multiple of 2.73; using such bounds would be far too conservative.

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